Cosmological anisotropy from non-comoving dark matter and dark energy

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We consider a cosmological model in which the two major fluid components of the Universe, dark energy and dark matter, flow with distinct four-velocities. This cosmological configuration is equivalent to a single anisotropic fluid, expanding with a four-velocity that is an appropriate combination of the two fluid four-velocities. The energy density of the single cosmological fluid is larger than the sum of the energy densities of the two perfect fluids, i.e., dark energy and dark matter, respectively, and contains a correction term due to the anisotropy generated by the differences in the four-velocities. Furthermore, the gravitational field equations of the two-fluid anisotropic cosmological model are obtained for a Bianchi type I geometry. By assuming that the non-comoving motion of the dark energy and dark matter induces small perturbations in the homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker type cosmological background, and that the anisotropy parameter is small, the equations of the cosmological perturbations due to the non-comoving nature of the two major components are obtained. The time evolution of the metric perturbations is explicitly obtained for the cases of the exponential and power law background cosmological expansion. The imprints of a non-comoving dark energy - dark matter on the Cosmic Microwave Background and on the luminosity distance are briefly discussed, and the temperature anisotropies and the quadrupole are explicitly obtained in terms of the metric perturbations of the flat background metric. Therefore, if there is a slight difference between the four-velocities of the dark energy and dark matter, the Universe would acquire some anisotropic characteristics, and its geometry will deviate from the standard FLRW one. In fact, the recent Planck results show that the presence of an intrinsic large scale anisotropy in the Universe cannot be excluded *a priori*, so that the model presented in this work can be considered as a plausible and viable working hypothesis.

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I. INTRODUCTION

The recently released Planck data of the 2.7 full sky survey [1–5] have shown a number of intriguing features, whose explanation will certainly require a deep change in our standard view of the Universe. These recent observations have measured the cosmic microwave background (CMB) to an unprecedented precision. One of the most striking results of the nominal mission is that the best-fit Hubble constant has the value $H_0 = 67.4 \pm 1.2$ km s^{-1}Mpc^{-1}, with a dark energy density parameter $\Omega_\Lambda = 0.686 \pm 0.020$, and a matter density parameter $\Omega_M = 0.307 \pm 0.019$. Even though generally the Planck data confirm the foundations of the ΛCDM (ΛCold Dark Matter) model, the observational data show some tension between the fundamental principle of the model and observations. For example, the Planck data combined with the WMAP polarization data show that the index of the power spectrum is given by $n_s = 0.9603 \pm 0.0073$ [2, 3], at the pivot scale $k_0 = 0.05$ Mpc^{-1}, which rules out the exact scale-invariance ($n_s = 1$) at more than 5σ. In addition to this, the joint constraints on r (tensor-to-scalar ratio) and n_s can constrain inflation models. For example, inflationary models with a power-law potential ϕ^4 cannot provide reasonable number of e-folds (between 50-60) in the restricted space of $r - n_s$ at around a 2σ level.

More interestingly, the search for the background geometry and topology of the Universe reveals that a Bianchi pattern, corresponding to an isotropic geometry of the Universe may account, in a quite efficient way, for some large-scale anomalies seen in the Planck data [5]. A Bayesian search for an anisotropic Bianchi VII$_h$ geometry was performed, by using the Planck data in [5]. In a non-physical setting, where the Bianchi parameters are decoupled from the standard cosmology, the observational data favour a Bianchi component with a Bayes factor of at least 1.5 units of log-evidence. However, in the physically motivated setting where the Bianchi parameters are fitted simultaneously with the standard cosmological parameters, no evidence for a Bianchi VII$_h$ cosmology was found [5].

According to the ΛCDM model, the material content of the Universe consists of two components: dark energy, and dark matter, respectively [6]. The observed late-time cosmic acceleration of the Universe [7] can be successfully explained by introducing either a fundamental cosmological constant Λ, which would represent an intrinsic curvature of space-time, or a dark energy, a hypothetical

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fluid component in the form of a zero-point-energy that pervades the whole Universe, which would mimic a cosmological constant (at least during the late stage of the cosmological evolution). Currently one of the main dark energy scenarios is based on the so-called *quintessence* [8], where dark energy corresponds to a scalar particle ϕ. There are also other dark energy candidates, such as the Chaplygin gas model [9], in which dark energy is a hypothetical substance that satisfies an exotic equation of state in the form $p = A/\rho^{-\alpha}$, where p is the pressure, ρ is the energy density, and A and α are positive constants. Modifying gravity at large scales, or considering extra-dimensional cosmological models can also provide an explanation of the late acceleration of the Universe [10, 22].

Dark matter is thought to be composed of cold neutral weakly interacting massive particles, beyond those existing in the Standard Model of Particle Physics, and not yet detected in accelerators or in direct and indirect searches. There are many possible candidates for dark matter, the most popular ones being the axions and the weakly interacting massive particles (WIMP) (for a review of the particle physics aspects of dark matter see [12]). Their interaction cross section with normal baryonic matter, while extremely small, are expected to be non-zero and we may expect to detect them directly. Scalar fields or other long range coherent fields coupled to gravity, have been considered as potential dark matter candidates [12]. Recently, the possibility that the galactic dynamics of massive test particles may be understood in the context of modified theories of gravity, without the need for dark matter, was also explored [13].

Now, precise observations of the cosmic microwave background radiation enable us to test the fundamental predictions of inflation on primordial fluctuations such as scale independence and Gaussianity [1–5]. The statistical isotropy has been a robust prediction protected by the cosmic no-hair conjecture, which claims that inflation washes out classical anisotropy. However, several recent observations of the large scale structure of the Universe have questioned the principles of homogeneity and isotropy [14]. As combined with the recent Planck results, these robust observational results show that the presence of an intrinsic large scale anisotropy in the Universe cannot be excluded *a priori*. Recently, the possibility that the geometry of the Universe may not be of the Friedmann-Lemaitre-Robertson-Walker (FLRW) type was investigated from different points of view [15, 16]. In particular, in [15] the homogeneous locally rotationally symmetric (LRS) class of metrics were considered as describing the global geometry of the Universe. Such spacetimes possess a preferred direction in the sky and at the same time a CMB which is isotropic at the background level. A distortion of the luminosity distances has been obtained, and used to test the model against the CMB and supernovae data. It has also been shown that the latter exhibit a higher-than-expected dependence on angular position. Local constraints from limits from simulated distance measurements for various distributions of survey fields in a Bianchi I anisotropic universe were obtained [17]. The effects of fitting for line of sight properties where isotropic dynamics is assumed were also considered, as well as the comparison of sensitivities of observational probes for anisotropies, from astrophysical systematics and from dark energy.

In addition to these, especially in the view of the latest Planck data, the empirical evidence for the existence of cosmological anisotropies is strong. But, its physical origin is still unknown, although the most popular explanation being that the primordial fluctuations are deviating from isotropy [1]. However, there is no clearly established physical mechanism that could lead to such deviations.

It is the purpose of the present paper to consider another possibility of explaining the presence of the anisotropies on a large cosmological scale, namely, the possibility that the two major components of the Universe, dark energy and dark matter, are not comoving. In standard cosmology the assumption that all constituents of the Universe move in the same frame, with the same four-velocity, is a fundamental (but not rigorously justifiable) assumption. This allows adopting a frame, which is comoving with matter in all its forms, in which the components of the four velocities u^μ of all components can be chosen as $u^\mu = (1, 0, 0, 0)$. Consequently, the global thermodynamic parameters of the Universe are just the sum of the individual thermodynamic parameters. Thus, for example, in the standard model the energy density of the Universe ρ is given by $\rho = \sum_{i=1}^n \rho_i$, where ρ_i, $i = 1, 2, .., n$ are the energy densities of the individual components.

However, there is no basic physical or observational principle that would require that all components in the Universe should have the same four-velocity, that is, one could describe the dynamics of the Universe in a single frame, comoving with all the constituent components. In this paper we will investigate the situation in which the Universe may be modeled as a mixture of two non-interacting perfect fluids, namely dark energy and dark matter, respectively, possessing different four-velocities. This configuration is formally equivalent to a single anisotropic fluid [18–21]. Therefore, if there is a slight difference between the four-velocities of the dark energy and dark matter, the Universe would acquire some anisotropic characteristics, and its geometry will deviate from the standard FLRW one.

The thermodynamic parameters (energy density and pressure) of the non-comoving two-fluid Universe are obtained by reducing the system to a single fluid, described by an anisotropic energy-momentum tensor. The energy density of the single fluid is greater than the sum of the energy densities of the dark energy and dark matter, respectively, and it contains a correction due to anisotropy. As a cosmological application of the model we consider an effective induced Bianchi type I geometry, generated by the non-comoving expansion of the dark energy and dark matter, respectively. The general properties of the

system are discussed, and the expression of some observationally important parameters (expansion and shear) are obtained. By using a simple perturbative approach, some simple cases of non-comoving dark anergy and dark matter cosmological models are constructed.

The possibility that there may exist different rest frames for dark matter and dark energy has been also investigated in [22], where it is shown that the existence of large scale bulk flows could be signaling the presence of moving dark energy at the time when photons decoupled from matter. Bianchi I type cosmological models, the evolution of the homogeneous perturbations up to second order, and the effects on the CMB quadrupole were also considered.

The possibilities of observationally testing the non-comoving dark energy-dark matter are briefly discussed. In particular we analyze the imprints of the anisotropy on the CMB spectrum, and on the luminosity distance. The expressions of the distribution of the temperature anisotropies and of the quadrupole C_2 are explicitly obtained in terms of the metric perturbations of the background flat geometry. This opens the possibility of observationally constraining the parameters of the model by using the recent Planck results [1], as well as the type Ia supernovae data [7].

The present paper is organized as follows. The reduction of a two-fluid system to a single effective anisotropic fluid description is discussed in Section II. The cosmological gravitational field equations of the model are obtained in Section III, where some general properties of the non-comoving two fluid Universe are also discussed. The evolution of the cosmological perturbations due to the non-comoving motion of the dark energy and dark matter are considered in Section IV. The observational implications of our results are investigated in Section V. We discuss and conclude our results in Section VI. Throughout this paper, we use the natural system of units with $c = 8\pi G = 1$.

II. ANISOTROPIC FLUID DESCRIPTION OF THE NON-COMOVING DARK ENERGY–DARK MATTER COSMOLOGICAL MODELS

We assume that the Universe consists of a mixture of two basic fluid components: dark energy, with energy density and pressure ρ_{DE} and p_{DE}, and four-velocity u_{DE}^{μ}, respectively; and dark matter, with thermodynamic parameters ρ_{DM} and p_{DM}, respectively, and four-velocity u_{DM}^{μ}. In standard cosmology it is assumed that the two cosmological fluids are comoving, which implies

$$u_{DE}^{\mu} \equiv u_{DM}^{\mu} \equiv u^{\mu}. \tag{1}$$

This condition allows to choose a comoving frame in the study of the cosmological dynamics, where the components of the four velocity of all components can be reduced to the form $u^{\mu} = (1, 0, 0, 0)$. However, in the present study we relax this condition, by assuming that, at least in some periods of the cosmological evolution, the two fundamental components of the Universe may have had different four-velocities, so that

$$u_{DE}^{\mu} \neq u_{DM}^{\mu}. \tag{2}$$

The physical and thermodynamical properties of the two-component cosmological fluid are described by the energy-momentum tensor $T^{\mu\nu}$ of the Universe, given by the sum of each individual components as

$$\begin{aligned} T^{\mu\nu} &= (\rho_{DE} + p_{DE}) u_{DE}^{\mu} u_{DE}^{\nu} - p_{DE} g^{\mu\nu} + \\ &\quad (\rho_{DM} + p_{DM}) u_{DM}^{\mu} u_{DM}^{\nu} - p_{DM} g^{\mu\nu}. \end{aligned} \tag{3}$$

The four-velocities of the dark energy and dark matter are normalized according to $u_{DE}^{\mu} u_{DE\,\mu} = 1$ and $u_{DM}^{\mu} u_{DM\,\mu} = 1$, respectively. In the following, we assume that the energy density and the pressure of the dark energy satisfy the conditions $\rho_{DE} + p_{DE} \neq 0$, that is, dark energy cannot be described by a "pure" cosmological constant.

The study of the cosmological models described by an energy-momentum tensor having the form given by Eq. (3) can be significantly simplified if we cast it into the standard form of perfect anisotropic fluids. This can be done by means of the transformations $u_{DE}^{\mu} \to u_{DE}^{*\mu}$ and $u_{DM}^{\mu} \to u_{DM}^{*\mu}$, respectively, so that [18–20]

$$\begin{pmatrix} u_{DE}^{*\mu} \\ u_{DM}^{*\mu} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sqrt{\frac{\rho_{DM} + p_{DM}}{\rho_{DE} + p_{DE}}}\sin\alpha \\ -\sqrt{\frac{\rho_{DE} + p_{DE}}{\rho_{DM} + p_{DM}}}\sin\alpha & \cos\alpha \end{pmatrix} \times \begin{pmatrix} u_{DE}^{\mu} \\ u_{DM}^{\mu} \end{pmatrix}, \tag{4}$$

represents a "rotation" of the four-vector velocities in the $(u_{DE}^{\mu}, u_{DM}^{\mu})$ velocity space. Explicitly, the transfor-

mations (4) take the form

$$u_{DE}^{\mu} \to u_{DE}^{*\mu} = u_{DE}^{\mu}\cos\alpha + \sqrt{\frac{\rho_{DM} + p_{DM}}{\rho_{DE} + p_{DE}}} u_{DM}^{\mu}\sin\alpha, \tag{5}$$

$$u_{DM}^{\mu} \rightarrow u_{DM}^{*\mu} = u_{DM}^{\mu} \cos\alpha - \sqrt{\frac{\rho_{DE} + p_{DE}}{\rho_{DM} + p_{DM}}} u_{DE}^{\mu} \sin\alpha, \tag{6}$$

respectively.

The velocity transformations given by Eq. (4) leave the quadratic form $(\rho_{DE} + p_{DE}) u_{DE}^{\mu} u_{DE}^{\nu} + (\rho_{DM} + p_{DM}) u_{DM}^{\mu} u_{DM}^{\nu}$ invariant. Thus,

$$T^{\mu\nu}\left(u_{DE}^{\mu}, u_{DM}^{\nu}\right) = T^{\mu\nu}\left(u_{DE}^{*\mu}, u_{DM}^{*\nu}\right). \tag{7}$$

Next, we choose $u_{DE}^{*\mu}$ and $u_{DM}^{*\mu}$ such that one becomes timelike, while the other is spacelike. Therefore, the two transformed four-vector velocities satisfy the condition $u_{DE}^{*\mu} u_{DM\,\mu}^{*} = 0$. With the use of Eqs. (5)-(6) and $U^{*\mu} W_{\mu}^{*} = 0$, we obtain first the rotation angle as

$$\tan 2\alpha = 2 \frac{\sqrt{(\rho_{DE} + p_{DE})(\rho_{DM} + p_{DM})}}{\rho_{DE} + p_{DE} - (\rho_{DM} + p_{DM})} u_{DE}^{\mu} u_{DM\,\mu}. \tag{8}$$

Note that if $\rho_{DE} + p_{DE} = 0$, which corresponds to the case of the cosmological constant, the rotation angle is $\alpha = 0$, and therefore the rotation in the velocity space reduces to the identical transformation.

Now, by defining the quantities

$$V^{\mu} = \frac{u_{DE}^{*\mu}}{\sqrt{u_{DE}^{*\alpha} u_{\alpha\,DE}^{*}}}, \tag{9}$$

$$\chi^{\mu} = \frac{u_{DM}^{*\mu}}{\sqrt{-u_{DM}^{*\alpha} u_{DM\,\alpha}^{*}}}, \tag{10}$$

and

$$\begin{aligned}\varepsilon &= T^{\mu\nu} V_{\mu} V_{\nu} \\ &= (\rho_{DE} + p_{DE}) u_{DE}^{*\alpha} u_{DE\,\alpha}^{*} - (p_{DE} + p_{DM}),\end{aligned} \tag{11}$$

$$\begin{aligned}\Psi &= T^{\mu\nu} \chi_{\mu} \chi_{\nu} \\ &= (p_{DE} + p_{DM}) - (\rho_{DM} + p_{DM}) u_{DM}^{*\alpha} W_{DM\,\alpha}^{*}\end{aligned} \tag{12}$$

$$\Pi = p_{DE} + p_{DM}, \tag{13}$$

respectively, the energy-momentum tensor of the two non-interacting perfect fluids can be written as

$$T^{\mu\nu} = (\varepsilon + \Pi) V^{\mu} V^{\nu} - \Pi g^{\mu\nu} + (\Psi - \Pi) \chi^{\mu} \chi^{\nu}, \tag{14}$$

where $V^{\mu} V_{\mu} = 1 = -\chi^{\mu} \chi_{\mu}$ and $\chi^{\mu} V_{\mu} = 0$ [18–20]. The energy-momentum tensor of the two-fluid cosmological model, in which the components have different four-velocities, given by Eq. (14), is the standard form for anisotropic fluids [20].

The energy density ε and the radial pressure Ψ of the anisotropic Universe are given by

$$\begin{aligned}\varepsilon = &\frac{1}{2}\left(\rho_{DE} + \rho_{DM} - p_{DE} - p_{DM}\right) + \\ &\frac{1}{2}\sqrt{\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right)^2 + 4\left(\rho_{DE} + p_{DE}\right)\left(\rho_{DM} + p_{DM}\right)\left[\left(u_{DE}^{\mu} u_{DM\,\mu}\right)^2 - 1\right]},\end{aligned} \tag{15}$$

and

$$\begin{aligned}\Psi = &-\frac{1}{2}\left(\rho_{DE} + \rho_{DM} - p_{DE} - p_{DM}\right) + \\ &\frac{1}{2}\sqrt{\left(\rho_{DE} + p_{DE} - \rho_{DM} - p_{DM}\right)^2 + 4\left(\rho_{DE} + p_{DE}\right)\left(\rho_{DM} + p_{DM}\right)\left(u_{DE}^{\mu} u_{DM\,\mu}\right)^2},\end{aligned} \tag{16}$$

respectively [18–20].

Since the two cosmological fluids, the dark energy, and the dark matter, have different four-velocities, we may write $u_{DE}^{\mu} u_{DM\,\mu} = 1 + b/2$, where generally b is an arbitrary function of the radial and time coordinates. The functional form of b can be obtained from Eq. (8), which

gives first

$$\begin{aligned}u_{DE}^{\mu} u_{DM\,\mu} = &\frac{1}{2}\tan 2\alpha\left[\sqrt{\frac{\rho_{DE} + p_{DE}}{\rho_{DM} + p_{DM}}} \right. \\ &\left. -\sqrt{\frac{\rho_{DM} + p_{DM}}{\rho_{DE} + p_{DE}}}\right],\end{aligned} \tag{17}$$

while b can be obtained as

$$b = \left[\sqrt{\frac{\rho_{DE} + p_{DE}}{\rho_{DM} + p_{DM}}} - \sqrt{\frac{\rho_{DM} + p_{DM}}{\rho_{DE} + p_{DE}}}\right]\tan 2\alpha - 2. \tag{18}$$

By assuming that the physical parameters of the dark energy and dark matter satisfy the condition

$$2b\left(1+\frac{b}{8}\right)\frac{(\rho_{DE}+p_{DE})(\rho_{DM}+p_{DM})}{(\rho_{DE}+\rho_{DM}+p_{DE}+p_{DM})^2}\ll 1,\quad (19)$$

the energy density, the radial and tangential pressures of the Universe can be obtained as

$$\varepsilon=\rho_{DE}+\rho_{DM}+\frac{b}{2}\left(1+\frac{b}{8}\right)\frac{(\rho_{DE}+p_{DE})(\rho_{DM}+p_{DM})}{(\rho_{DE}+\rho_{DM}+p_{DE}+p_{DM})},\tag{20}$$

$$\Psi=p_{DE}+p_{DM}+\frac{b}{2}\left(1+\frac{b}{8}\right)\frac{(\rho_{DE}+p_{DE})(\rho_{DM}+p_{DM})}{(\rho_{DE}+\rho_{DM}+p_{DE}+p_{DM})},\tag{21}$$

and

$$\Pi=p_{DE}+p_{DM},\tag{22}$$

respectively. As one can readily verify from Eq. (20), the energy density of the non-comoving two fluid Universe is different from the sum of the energy densities of the component fluids.

III. ANISOTROPIC TWO FLUID COSMOLOGICAL DYNAMICS

In the present Section, we consider some cosmological applications of the non-comoving dark energy-dark matter cosmological model. As a first step in our analysis, we write down the gravitational field equations corresponding to the anisotropic evolution of the Universe. The general properties of the model are analyzed, and it is shown that the non-comoving nature of the cosmological dynamics affects the observational parameters of the Universe, by inducing some specific anisotropic effects in the evolution of both expansion and shear parameters. Finally, an explicit dark energy model is considered, and the general solution of the field equations, describing the dark energy-dark matter mixture is obtained.

A. Gravitational field equations of the two non-comoving fluid cosmological model

For the two fluid energy-momentum tensor, with components given by Eqs. (14), the Einstein gravitational field equations can be written as [19]

$$R_{\mu\nu}V^\mu V^\nu=\frac{1}{2}\left(\varepsilon+2\Pi+\psi\right),\tag{23}$$

$$R_{\mu\nu}V^\mu h^\nu_\sigma=0,\tag{24}$$

$$R_{\mu\nu}h^\mu_\sigma h^\nu_\lambda=\frac{1}{2}\left[\varepsilon-\frac{1}{3}\left(2\Pi+\Psi\right)\right]h_{\sigma\lambda}+(\Psi-\Pi)\left(\chi_\sigma\chi_\lambda-\frac{1}{3}h_{\sigma\lambda}\right),\tag{25}$$

where $h^{\mu\nu}=g^{\mu\nu}-V^\mu V^\nu$. The conservation of the energy-momentum tensor $\nabla_\mu T^\mu_\nu=0$ yields

$$\dot{\varepsilon}+(\varepsilon+\Pi)\nabla_\mu V^\mu-(\Psi-\Pi)\chi'^\mu V_\mu=0,\tag{26}$$

$$(\varepsilon+\Pi)\dot{V}^\mu+h^{\mu\nu}\nabla_\nu\Pi+(\Psi-\Pi)'\chi^\mu+(\Psi-\Pi)\left(\nabla_\nu\chi^\nu\chi^\mu+\chi'_\nu h^{\nu\mu}\right)=0,\tag{27}$$

where the overdot and the prime are defined as $\dot{()}=\nabla_\mu()V^\mu$, and $()'=\nabla_\mu()\chi^\mu$, respectively.

Since the two-fluid dark energy-dark matter cosmological mixture represents an anisotropic fluid, on a cosmological scale the corresponding geometry is also anisotropic. In comoving Cartesian coordinates $x^0=t$, $x^1=x$, $x^2=y$, and $x^3=z$ we may choose $V^1=V^2=V^3=0$, $V^0V_0=1$, and $\chi^0=\chi^1=\chi^2=0$, $\chi^3\chi_3=-1$, respectively [18–20]. Therefore the components of the energy-momentum of two non-comoving cosmological perfect fluids take the form

$$T^0_0=\varepsilon,\qquad T^1_1=T^2_2=-\Pi,\qquad T^3_3=-\Psi,\tag{28}$$

where ε is the total energy-density of the mixture of fluids, $\Psi=P_z$ is the pressure along the z direction, while $\Pi=P_x=P_y$ is the pressure along the x and y directions. The energy-momentum tensor components given by Eqs. (28) shows that the pressure Ψ along the $z-$direction may be different with respect to the pressure Π on the $x-y$ plane.

In a homogeneous Universe with a two-fluid mixture the simplest geometry exhibiting this symmetry of the energy-momentum tensor is the flat Bianchi type I geometry, with the line element given by

$$ds^2=dt^2-a_1^2(t)dx^2-a_2^2(t)dy^2-a_3^2(t)dz^2,\tag{29}$$

where $a_i(t)$ $(i=1,2,3)$ are the directional scale factors. In this geometry the gravitational field equations take the form

$$3\dot{H}+H_1^2+H_2^2+H_3^2=-\frac{1}{2}\left(\varepsilon+\Psi+2\Pi\right),\tag{30}$$

$$\frac{1}{V}\frac{d}{dt}(VH_i)=\frac{1}{2}(\varepsilon-\Pi),\qquad i=1,2,\tag{31}$$

and

$$\frac{1}{V}\frac{d}{dt}(VH_3)=\frac{1}{2}(\varepsilon-\Psi),\tag{32}$$

respectively. We have denoted

$$V=a_1a_2a_3,\qquad H_i=\frac{\dot{a}_i}{a_i},\qquad i=1,2,3\tag{33}$$

and

$$H=\frac{1}{3}\left(\sum_{i=1}^3 H_i\right)=\frac{\dot{V}}{3V}\tag{34}$$

respectively. The expansion parameter θ is given by $\theta = 3H$. From the conservation of the total energy-momentum tensor of the two-fluid cosmological model we obtain the evolution equation of the energy density of the system, given by

$$\dot{\varepsilon} + 3(\varepsilon + \Pi)H - (\Psi - \Pi)H_3 = 0. \qquad (35)$$

B. General properties of the anisotropic Bianchi I two fluid cosmological model

From Eqs. (31) it follows that for the considered form of the anisotropic energy-momentum tensor one can take $a_1(t) = a_2(t)$ without any loss of generality. Therefore the expansion parameter of the Universe can be obtained as

$$\theta = 2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}, \qquad (36)$$

while the shear scalar σ of the Universe is given by

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right). \qquad (37)$$

Equations (36) and (37) allow the determination of the directional Hubble parameters H_1 and H_3 in terms of observable cosmological parameters as

$$H_1 = \frac{\theta}{3} - \frac{1}{\sqrt{3}}\sigma, \qquad H_3 = \frac{\theta}{3} + \frac{2}{\sqrt{3}}\sigma, \qquad (38)$$

respectively.

The large-time behavior of an anisotropic cosmological model can be obtained from the study of the anisotropy parameter A, defined by

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \qquad (39)$$

If $A = 0$ the cosmological model is isotropic. The anisotropic cosmological pressure differences can be obtained, in terms of the observable quantities, as

$$\Psi - \Pi = \sqrt{3}\sigma + \theta\sigma, \qquad (40)$$

or, equivalently,

$$\sqrt{3}\sigma + \theta\sigma = \frac{1}{2}\left[-(\rho_{DE} + \rho_{DM}) + \sqrt{(\rho_{DE} + p_{DE} - \rho_{DM} - p_{DM})^2 + 4(\rho_{DE} + p_{DE})(\rho_{DM} + p_{DM})(u_{DE}^\mu u_{DM\,\mu})^2}\right]. \qquad (41)$$

By assuming that the parameter $\beta(t)$, describing the deviations from a comoving motion, is small, then we obtain

$$\beta(t) \equiv \frac{(\rho_{DE} + p_{DE})(\rho_{DM} + p_{DM})(u_{DE}^\mu u_{DM\,\mu})^2}{(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM})^2} \ll 1. \qquad (42)$$

To first order in β, we obtain the following expressions of the energy density and z-direction pressure of the two non-comoving fluid cosmological model,

$$\varepsilon \approx \rho_{DE} + \rho_{DM} + \beta(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}), \qquad (43)$$

and

$$\Psi = p_{DE} + p_{DM} + \beta(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}), \qquad (44)$$

respectively. Therefore the total energy density and pressure of the two fluid cosmological model is not equal to the sum of the energy densities and pressures of the individual matter components of the system.

By adding Eqs. (31) and (32) we obtain

$$\dot{H} + 3H^2 = \frac{1}{2}\varepsilon - \frac{1}{3}\left(\Pi + \frac{\Psi}{2}\right), \qquad (45)$$

or, equivalently,

$$\ddot{V} + \left(\Pi + \frac{1}{2}\Psi - \frac{3}{2}\varepsilon\right)V = 0. \qquad (46)$$

IV. PERTURBATIVE APPROACH TO NON-COMOVING TWO FLUID COSMOLOGICAL MODELS

In the following, we consider a simple perturbative solution of the field equations (30)-(32) by assuming that the anisotropic expansion due to the non-comoving motion of the dark energy and dark matter represents a small perturbation of the background isotropic, flat FLRW geometry, described by the isotropic scale factor $a(t)$. We also assume that the expansion rates along the x and y axes are the same, so that $a_1(t) = a_2(t)$, and the perturbations along these axes are identical. Therefore the scale factor of the perturbed Bianchi type I geometry can be given as

$$a_i(t) = a(t) + \delta_i(t), \qquad i = 1, 2, 3, \qquad (47)$$

where $\delta_i(t)$, $i = 1, 2, 3$, are small corrections in the scale factor $a(t)$ of the isotropic Universe, induced by the small

differences between the four-velocities of the dark matter and dark energy. Moreover, we assume that

$$\delta_1(t) = \delta_2(t). \tag{48}$$

The Hubble parameter of the background isotropic space is $H_0(t) = \dot{a}/a$. For the directional Hubble parameters of the anisotropic expansion we obtain

$$H_i(t) = \frac{\dot{a}(t) + \dot{\delta}_i(t)}{a(t) + \delta_i(t)} \approx H_0(t)\left[1 - \frac{\delta_i(t)}{a(t)}\right] + \frac{\dot{\delta}_i(t)}{a(t)}, \tag{49}$$

where $i = 1, 2, 3$. Thus, the mean Hubble factor can be obtained as

$$H(t) = \frac{1}{3}\left[3H_0(t) - H_0(t)\frac{\delta(t)}{a(t)} + \frac{\dot{\delta}(t)}{a(t)}\right], \tag{50}$$

where we have denoted

$$\delta(t) = 2\delta_1(t) + \delta_3(t), \tag{51}$$

while the square of the directional Hubble factors is given by

$$H_i^2(t) \approx H_0^2(t) - 2H_0^2(t)\frac{\delta_i(t)}{a(t)} + 2H_0(t)\frac{\dot{\delta}_i(t)}{a(t)}, \tag{52}$$

where $i = 1, 2, 3$.

For the volume $V(t)$ of the Universe we obtain

$$V(t) = a^3(t)\left[1 + \frac{\delta(t)}{a(t)}\right], \tag{53}$$

and $1/V(t) \approx 1/a^3(t)$.

Hence, in the first order approximation, the gravitational field equations describing the slightly perturbed isotropic flat FLRW Universe become

$$3\dot{H}_0(t) + 3H_0^2(t) - \left[\dot{H}_0(t) + H_0^2(t)\right]\frac{\delta(t)}{a(t)} + \frac{\ddot{\delta}(t)}{a(t)} =$$
$$-\frac{1}{2}\Big[\rho_{DE} + \rho_{DM} + 3\left(p_{DE} + p_{DM}\right)$$
$$+ 2\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right)\Big], \tag{54}$$

$$\frac{1}{a^3(t)}\frac{d}{dt}\left[a^3(t)H_0(t)\right] + \frac{1}{a^3(t)}\frac{d}{dt}\left[a^2(t)H_0(t)\delta_+(t)\right]$$
$$+\frac{1}{a^3}\frac{d}{dt}\left[a^2(t)\dot{\delta}_1(t)\right] = \frac{1}{2}\Big[\rho_{DE} - p_{DE} + \rho_{DM} - p_{DM}$$
$$+\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right)\Big], \tag{55}$$

and

$$\frac{1}{a^3(t)}\frac{d}{dt}\left[a^3(t)H_0(t)\right] + \frac{2}{a^3}\frac{d}{dt}\left[a^2(t)H_0(t)\delta_1(t)\right]$$
$$+\frac{1}{a^3}\frac{d}{dt}\left[a^2(t)\dot{\delta}_3(t)\right] = \frac{1}{2}\left(\rho_{DE} - p_{DE} + \rho_{DM} - p_{DM}\right), \tag{56}$$

where we have denoted $\delta_+(t) = \delta_1(t) + \delta_2(t)$. Equation (46), giving the time evolution of the volume of the Universe, takes the form

$$\frac{d^2}{dt^2}a^3(t) + \frac{3}{2}\left[a^3(t)\left(p_{DE} - \rho_{DE} + p_{DM} - \rho_{DM}\right)\right] +$$
$$\frac{d^2}{dt^2}a^2(t)\delta(t) + \frac{3}{2}\left[a^2(t)\left(p_{DE} - \rho_{DE} + p_{DM} - \rho_{DM}\right)\right] \times$$
$$\delta(t) - \frac{1}{2}\left[a^3(t)\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right)\right] = 0. \tag{57}$$

The gravitational field equations of the background, isotropic geometry, are the standard Friedmann equations, determining the scale factor a as

$$3\frac{\dot{a}^2(t)}{a^2(t)} = \rho_{DE} + \rho_{DM}, \tag{58}$$

$$2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a(t)} = -p_{DE} - p_{DM}. \tag{59}$$

respectively. With the use of Eqs. (58) and (59) we immediately obtain

$$\frac{1}{a^3(t)}\frac{d\left[a^3(t)H_0(t)\right]}{dt} = \frac{1}{2}\left(\rho_{DE} - p_{DE} + \rho_{DM} - p_{DM}\right), \tag{60}$$

$$3\dot{H}_0(t) + 3H_0^2(t) = -\frac{1}{2}\left[\rho_{DE} + \rho_{DM} + 3\left(p_{DE} + p_{DM}\right)\right], \tag{61}$$

and

$$\frac{d^2a^3(t)}{dt^2} + \frac{3}{2}\left[a^3(t)\left(p_{DE} - \rho_{DE} + p_{DM} - \rho_{DM}\right)\right] = 0, \tag{62}$$

respectively.

Therefore, the gravitational field equations describing the time evolution of a cosmological model with a small anisotropy in the z-direction, induced by the non-comoving nature of the dark energy and dark matter, take the form

$$\frac{\ddot{\delta}(t)}{a(t)} - \frac{\ddot{a}(t)}{a(t)}\frac{\delta(t)}{a(t)} = -\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right), \tag{63}$$

$$\frac{1}{a^3(t)}\frac{d}{dt}\left[a^2(t)H_0(t)\delta_+(t)\right] + \frac{1}{a^3}\frac{d}{dt}\left[a^2(t)\dot{\delta}_1(t)\right] =$$
$$\frac{1}{2}\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right), \tag{64}$$

$$\frac{2}{a^3}\frac{d}{dt}\left[a^2(t)H_0(t)\delta_1(t)\right] + \frac{1}{a^3}\frac{d}{dt}\left[a^2(t)\dot{\delta}_3(t)\right] = 0. \tag{65}$$

Equations (63)-(65) represent a system of three differential equations for the three unknown functions $(\delta_1(t), \delta_3(t), \beta(t))$.

The deceleration parameter q of the non-comoving two fluid anisotropic Universe is defined as

$$q = \frac{d}{dt}\frac{1}{H(t)} - 1, \qquad (66)$$

and for the case of the small deviation from isotropy it is given by

$$q = q_0 + \frac{1}{3}\frac{d}{dt}\frac{1}{aH_0}\left[\delta(t) - \frac{\dot{\delta}(t)}{H_0}\right], \qquad (67)$$

where $q_0 = d\left[1/H_0(t)\right]/dt - 1 = -a(t)\ddot{a}(t)/\dot{a}^2(t)$ is the deceleration parameter of the isotropic FLRW Universe.

Equation (65) can be immediately integrated to give

$$2H_0(t)\delta_1(t) + \dot{\delta}_3(t) = \frac{C}{a^2(t)}, \qquad (68)$$

where C is an arbitrary constant of integration. In order to determine the value of C we assume that at $t = t_0$ the Universe was in an approximate isotropic state, so that $H_i(t_0) \approx H_0(t_0)$, $i = 1, 2, 3$, with a small anisotropic perturbation $\delta_i(t_0) = \delta_{0i}$, $i = 1, 2, 3$. Then from Eq. (49) we have $\dot{\delta}_i(t_0) = H_0(t_0)\delta_{0i}$, $i = 1, 2, 3$. For the initial values of δ we obtain $\delta(t_0) = 2\delta_1(t_0) + \delta_3(t_0) = \delta_0$, while $\dot{\delta}(t_0) = H_0(t_0)\delta_0$. Hence the value of the integration constant C is given by

$$\begin{aligned} C &= a^2(t_0)\left[2H_0(t_0)\delta_{01} + H_0(t_0)\delta_{03}\right] \\ &= a^2(t_0)H_0(t_0)\delta_0. \end{aligned} \qquad (69)$$

By multiplying Eq. (64) by two, and adding the resulting equation to Eq. (65), we obtain

$$\frac{2}{a^3(t)}\frac{d}{dt}\left[a^2(t)H_0(t)\delta(t)\right] + \frac{1}{a^3}\frac{d}{dt}\left[a^2(t)\dot{\delta}(t)\right] =$$
$$\beta\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right). \qquad (70)$$

By eliminating the term containing the energy densities between Eqs. (63) and (70), we obtain the second order differential equation governing the behavior of the total perturbation δ due to the non-comoving motion of the dark energy and dark matter as

$$2\ddot{\delta}(t) + 4H_0(t)\dot{\delta}(t) + \left[\dot{H}_0(t) + 3H_0^2(t)\right]\delta(t) = 0. \qquad (71)$$

Equation (71) must be integrated together with the initial conditions

$$\delta(t_0) = \delta_0, \dot{\delta}(t_0) = H_0(t_0)\delta_0. \qquad (72)$$

The behavior of the perturbations to isotropy induced by the non-comoving flow of the dark energy and dark matter is determined by the Hubble parameter of the

isotropic FLRW background. Once the function $\delta(t)$ is known, the time evolution of the perturbations along the $z-$axis is determined by the first order linear differential equation

$$\dot{\delta}_3(t) - H_0\delta_3(t) = \frac{C}{a^2(t)} - H_0\delta(t), \qquad (73)$$

with the general solution given by

$$\delta_3(t) = a(t)\left\{C_1 + \int\left[\frac{C}{a^3(t)} - \frac{H_0(t)}{a(t)}\delta(t)\right]dt\right\}, \quad (74)$$

where C_1 is an arbitrary integration constant to be determined from the initial condition $\delta_3(t_0) = \delta_{03}$. The perturbation from isotropy along the x and y axes can be obtained from

$$\delta_1(t) = \frac{1}{2}\left[\delta(t) - \delta_3(t)\right], \qquad (75)$$

while the anisotropy parameter β can be obtained as

$$\beta(t) = \frac{2H_0(t)\dot{\delta}(t) + \left[3\dot{H}_0(t)/2 + 5H_0^2(t)/2\right]\delta(t)}{a\left(\rho_{DE} + p_{DE} + \rho_{DM} + p_{DM}\right)}. \qquad (76)$$

Hence we have obtained the general solution of the perturbed gravitational field equations due to the non-comoving motion of the dark energy and dark matter.

As simple applications of the present formalism we will consider two cases, namely, the case of the constant H_0, corresponding to an exponentially expanding Universe, $a(t) = \exp(H_0 t)$, and of the power law expanding isotropic FLRW Universe, with $a(t) = t^n$. Note, however, that in order to obtain the expression of the anisotropy parameter β, the energy densities and pressures of the dark energy and dark matter must also be known.

A. Specific solution I: $a(t) = \exp(H_0 t)$

If $H_0 = $ constant, then the general solution of Eq. (71) is given by

$$\delta(t) = e^{-H_0 t}\left[C_2\cos\left(\sqrt{8}H_0 t\right) + C_3\sin\left(\sqrt{8}H_0 t\right)\right], \qquad (77)$$

where C_2 and C_3 are arbitrary constants of integration to be determined from the initial conditions $\delta(t_0) = \delta_0$, and $\dot{\delta}(t_0) = H_0(t_0)\delta_0$, respectively. For an exponentially expanding background Universe the magnitude of the total anisotropy δ decreases exponentially in time.

By using these expressions of $\delta(t)$ the perturbations $\delta_1(t)$ and $\delta_3(t)$ along the x, y and z axes can be explicitly computed for these particular cosmological models. For the de Sitter-type background expansion, we obtain

$$\delta_3(t) = C_1 e^{H_0 t} + \frac{e^{-2H_0 t} \left\{ H_0 e^{H_0 t} \left[\left(C_3 - \sqrt{2} C_2 \right) \sin \left(2\sqrt{2} H_0 t \right) + \left(C_2 + \sqrt{2} C_3 \right) \cos \left(2\sqrt{2} H_0 t \right) \right] - 2C \right\}}{6 H_0}, \qquad (78)$$

and

$$\delta_1(t) = \frac{e^{-2H_0 t} \left\{ 2 \left(C - 3C_1 H_0 e^{3H_0 t} \right) + H_0 e^{H_0 t} \left[\left(5C_2 - \sqrt{2} C_3 \right) \sin \left(2\sqrt{2} H_0 t \right) + \left(5C_2 + \sqrt{2} C_3 \right) \cos \left(2\sqrt{2} H_0 t \right) \right] \right\}}{12 H_0}, \qquad (79)$$

respectively. The time evolution of the anisotropic perturbation along the z−axis is proportional to the scale factor of the isotropic background, and thus for a de Sitter-type Universe the anisotropy in the z direction will increase in time, making the Universe more and more anisotropic. On the other hand, since in the large time limit δ tends to zero, $\lim_{t \to \infty} \delta = 0$, from Eq. (75) it follows that $\lim_{t \to \infty} \delta_1(t) = -(1/2) \lim_{t \to \infty} \delta_3(t) = -(1/2) \lim_{t \to \infty} C_1 \exp(H_0 t)$. However, in these large time limits the formalism developed in the present paper may not be applicable, since all the presented results have been derived under the assumption $\delta_i \ll a$, $i = 1, 2, 3$.

B. Specific solution II: power law expansion

For a power law expansion, with the scale factor of the form $a(t) = t^n$, Eq. (71) becomes

$$2\ddot{\delta} + \frac{4n}{t} \dot{\delta} + \frac{n(3n-1)}{t^2} \delta(t) = 0, \qquad (80)$$

with the general solution given by

$$\delta(t) = C_4 t^{r_1} + C_5 t^{r_2}, \qquad (81)$$

where

$$r_{1,2} = \frac{1 - 2n \pm \sqrt{2}\sqrt{2 - n - n^2}}{2}. \qquad (82)$$

In the case of the power law background expansion we obtain

$$\delta_3(t) = C_1 t^n + t \left(\frac{c t^{-2n}}{1 - 3n} + \frac{C_4 H_0 t^{r_1}}{n - r_1 - 1} + \frac{C_5 H_0 t^{r_2}}{n - r_2 - 1} \right), \qquad (83)$$

while for $\delta_1(t)$, we have

$$2\delta_1(t) = C_4 t^{r_1} - C_1 t^n + + C_5 t^{r_2} + C \frac{t^{1-2n}}{3n - 1}$$
$$+ C_4 \frac{H_0 t^{r_1 + 1}}{-n + r_1 + 1} + C_5 \frac{H_0 t^{r_2 + 1}}{-n + r_2 + 1}. \qquad (84)$$

The arbitrary integration constants C_4 and C_5 can be determined again from the initial conditions $\delta(t_0) = \delta_0$, and $\dot{\delta}(t_0) = H_0(t_0) \delta_0$, respectively.

V. OBSERVATIONAL IMPRINTS ON THE CMB AND SUPERNOVAE LUMINOSITIES OF THE NON-COMOVING DARK MATTER–DARK ENERGY MODEL

In the present Section, we briefly consider the observational implications of the non-comoving dark energy and dark matter models. The anisotropic expansion of the Universe would leave its imprints on both CMB and supernovae date. The amount of possible anisotropy existing in the Universe can be constrained by using the CMB and SNIa observations. Usually the present CMB data constraints can be much tighter than the constraints from the anisotropies in the luminosity–distance–redshift relationship of SNIa [23]. However, there are anisotropic models which can avoid this bound completely [23]. Within these bounds, the existence of anisotropy in the Universe can provide a potential explanation of various anomalies in the cosmological observations, especially in the CMB at the largest angles.

A. Effects on CMB on the non-comoving motion of dark energy and dark matter

A possible anisotropic expansion of the Universe will leave its imprint in the CMB. To calculate the CMB spectrum in a background geometry with a Bianchi type I metric given by Eq. (29) we follow the approach introduced in [23]. Photons emitted by astrophysical sources at cosmological distances move in the Bianchi type I Universe along the geodesic lines, given by

$$\frac{du^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0, \qquad (85)$$

where λ is an affine parameter, and the Christoffel symbols $\Gamma^\mu_{\alpha\beta}$ associated to the metric Eq. (29) are given by $\Gamma^0_{ii} = a_i^2 H_i$, and $\Gamma^i_{0i} = H_i$, $i = 1, 2, 3$ (no summation upon i in the Christoffel symbols).

The four-velocity $u^\mu = dx^\mu / d\lambda$ of the photons satisfy the normalization condition $u^\mu u_\mu = 0$, giving $(u_0)^2 = a_i^2 (u^i)^2$. Consider two photons, emitted at two time interval $(t_0 = t_e, t_1 = t_e + \tau)$, where $\tau \ll t_e$. By taking the difference of the two normalization conditions for photons in the first order of approximation in τ we obtain

[23]

$$u^0 \frac{d\tau}{d\lambda} = \sum_{i=1}^{3} a_i \dot{a}_i \left(u^i\right)^2 \tau(\lambda) + O\left(\tau^2\right). \qquad (86)$$

By introducing the redshift z, defined as $1 + z\left(\lambda_e\right) = \tau\left(\lambda_r\right)/\tau\left(\lambda_e\right)$, where $\tau\left(\lambda_r\right)$ is the time difference of the received signals, with the use of Eq. (86) we immediately obtain

$$\frac{d}{d\lambda} \ln(1+z) = \frac{1}{u^0} \sum_{i=1}^{3} a_i \dot{a}_i \left(u^i\right)^2. \qquad (87)$$

The photon velocity components u^i, $i = 1,2,3$, can be obtained from the photon geodesic equation of motion,

$$\frac{du^i}{d\lambda} + 2\frac{\dot{a}_i}{a_i} u^i u^0 = 0, \qquad i = 1,2,3, \qquad (88)$$

giving

$$u^i(t) = \frac{u_{0i}}{a_i^2(t)}, \qquad i = 1,2,3, \qquad (89)$$

where u_{0i}, $i = 1,2,3$ are arbitrary constants of integration. In order to determine the integration constants, we adopt the convention according to which the present day $t = t_0$ value of the scale factors is $a_i\left(t_0\right) = 1$. We denote the present day values of the photon four-velocity as as $u^i\left(t_0\right) = \hat{u}^i$, $i = 1,2,3$. Since one can always reparameterize the affine parameter λ without modifying the physical results, we can normalize the present day photon four-velocities so that $\sum_{i=1}^{3}\left(\hat{u}^i\right)^2 = 1$. Hence one may interpret the unit vector $\hat{u}$ in terms of the angles $(\hat{u}_x, \hat{u}_y, \hat{u}_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, representing the present day arrival angles of the photons to the observer. Therefore, by substituting the photon velocities into the redshift equation, we obtain [23]

$$1 + z\left(\hat{\vec{u}}\right) = \sqrt{\sum_{i=1}^{3} \frac{\hat{u}_i^2}{a_i^2}}, \qquad (90)$$

which can be rewritten as

$$1 + z\left(\hat{\vec{u}}\right) = \frac{1}{a_1}\sqrt{1 + \hat{u}_y^2 e_y^2 + \hat{u}_z^2 e_z^2}, \qquad (91)$$

where the eccentricities e_y^2 and e_z^2 are defined as

$$e_y^2 = \left(\frac{a_1}{a_2}\right)^2 - 1, \qquad e_z^2 = \left(\frac{a_1}{a_3}\right)^2 - 1. \qquad (92)$$

Since in the non-comoving dark energy-dark matter model we have considered we have assumed that $a_1 = a_2$, we obtain $e_y^2 = 0$, and

$$e_z^2 = \frac{2}{a}\left[\delta_1(t) - \delta_3(t)\right], \qquad (93)$$

respectively.

The temperature distribution is determined by the relation

$$T\left(\hat{\vec{u}}\right) = \frac{T_*}{1 + z\left(\hat{\vec{u}}\right)}, \qquad (94)$$

where the last scattering temperature T_* does not depend on the direction. However, due the anisotropic nature of the geometry of the Universe, photons coming from different directions will be redshifted by different amounts. The spatial average $\bar{T}$ of the temperature field is obtained as $4\pi\bar{T} = \int T\left(\hat{\vec{u}}\right) d\Omega_{\hat{\vec{u}}}$, while the anisotropies in the temperature field are given by

$$\begin{aligned} \delta T\left(\hat{\vec{u}}\right) &= 1 - \frac{T\left(\hat{\vec{u}}\right)}{\bar{T}} \\ &= 1 - \frac{T_*}{\bar{T}} \frac{1}{\sqrt{1 + 2\left[\delta_1\left(t_0\right) - \delta_3\left(t_0\right)\right]\cos^2\theta}} \end{aligned} \qquad (95)$$

The multipole spectrum C_l can be described by the coefficients in the spherical expansion of the temperature anisotropy field. The quadrupole Q_2 can be obtained as [23]

$$\begin{aligned} C_2 &= \frac{2}{5\sqrt{3}}\sqrt{e_z^4 + e_y^4 - e_z^2 e_y^2} = \frac{2}{5\sqrt{3}}e_z^2 \\ &= \frac{4}{5\sqrt{3}}\frac{1}{a}\left[\delta_1(t) - \delta_3(t)\right]. \end{aligned} \qquad (96)$$

The nominal value of the best fit quadrupole measured by Planck is $[2(2+1)/2\pi]C_2 = 299.495 \times 10^{-12}$ [2]. Therefore the recent observational results can be used to constrain the parameters of the non-comoving dark energy–dark matter model.

B. Constraints on non-comoving dark matter and dark energy from supernova observations

Anisotropy in the expansion rates of the Universe would, in principle, determine rotationally non-invariant luminosity distance–redshift relationships for the Ia type supernovae. The luminosity–redshift relationship of the SNIa can be used to probe the possible anisotropies in the expansion history of the Universe [23, 24]. The spatial geodesic Eq. (88) shows that the direction of a photon $\hat{\vec{u}}$ that is coming towards us from an astrophysical source is constant. This follows from the orthogonality property of the Bianchi I models. The null geodesic equation can be rewritten as

$$dt^2 = \left(\hat{u}_x^2 a_1^2 + \hat{u}_y^2 a_2^2 + \hat{u}_z^2 a_3^2\right) dr^2, \qquad (97)$$

where $r^2 = x^2 + y^2 + z^2$. The conformal distance can be obtained as $\int dr$. The luminosity distance at the redshift

z in the direction $\hat{u}$ is thus given by [23, 24]

$$d_L\left(z,\hat{u}\right) = (1+z) \int_{t_0}^{t(z)} \frac{dt}{\sqrt{\hat{u}_x^2 a_1^2 + \hat{u}_y^2 a_2^2 + \hat{u}_z^2 a_3^2}}. \quad (98)$$

Therefore high precision SNIa data can be used to constrain the non-comoving dark energy–dark matter models.

VI. DISCUSSIONS AND FINAL REMARKS

In the present paper, we have considered the theoretical possibility that our Universe may be modeled as a mixture of two non-interacting perfect fluids, namely, dark energy and dark matter, respectively, with different four-velocities. This model is formally equivalent to a Universe filled with a single anisotropic fluid, with different pressure components along the coordinate axes. The gravitational equations describing the non-comoving dark energy and dark matter mixture were explicitly obtained for a Bianchi type I geometry. By assuming that the difference in the four-velocities of the dark energy and dark matter is small, one can consider that the deviations from isotropy are also small, representing just a small perturbation of the homogeneous and isotropic background metric. In particular, the perturbation equations have been explicitly obtained and solved, for the case of the Bianchi type I geometry. Two simple cases, corresponding to an exponential and power law expansion of the background geometry have also been analyzed in detail.

Therefore, if there is a slight difference between the four-velocities of the dark energy and dark matter, the Universe would acquire some anisotropic characteristics, and its geometry will deviate from the standard FLRW one. In fact, the recent Planck results show that the presence of an intrinsic large scale anisotropy in the Universe cannot be excluded *a priori*, so that the model presented in this work can be considered as a plausible and viable working hypothesis. However, in the present paper we have considered that the deviations from isotropy are small, and we have restricted our analysis to the linear perturbation regime. If the amplitude of the perturbations does not satisfy the condition $\delta_i \ll a$, $i = 1, 2, 3$, then non-linear effects should also be taken into account, and incorporated into the description of the anisotropic effects generated by the non-comoving dark energy and dark matter.

We have also pointed out some possibilities of observationally testing the model, by using the recently released Planck data [1], as well as the type Ia supernovae data [7]. In the case of the CMB we have explicitly obtained the distribution of the temperature anisotropy and of the quadrupole C_2 as functions of the deviations δ_1 and δ_3 from the isotropic flat geometry. Once the form of the background FLRW geometry, more exactly the form of the scale factor $a(t)$ is known, one can use the obtained

relations to put tight constraints on the parameters of the non-comoving dark energy–dark matter model. Similar constraints can be obtained from the study of the anisotropic luminosity distance by using the type Ia supernovae observational data.

In this context, we emphasize that a major challenge for cosmology is to elucidate the nature of dark energy, which drives the accelerated expansion of the Universe. The simplest explanation for dark energy is the cosmological constant, where the latter has an equation of state $w = p_{DE}/\rho_{DE} = -1$. Scalar fields, alternatives to the cosmological constant, have an equation of state with $w \geq -1$. However, the CMB data alone does not strongly constrain w, due to the two-dimensional geometric degeneracy in the theoretical models. The recent results of Planck, combined with the supernova data, provide $w = -1.09 \pm 0.17$, or $w = -1.13^{+0.13}_{-0.14}$, generally in a good agreement with the cosmological constant hypothesis. However, at the present level of observational precision one cannot rule out the existence of a dark energy component of the Universe, distinct from a cosmological constant. For the case of a "pure" cosmological constant, the formalism developed in the present paper cannot be applied, since there is no flow associated to Λ.

In the case of the scalar field ϕ, with energy-momentum tensor

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\nabla^\lambda\phi\nabla_\lambda\phi/2 - V(\phi)g_{\mu\nu}, \quad (99)$$

where $V(\phi)$ is the scalar field potential, a perfect fluid description corresponding to an energy-momentum tensor $T_{DE\,\mu\nu} = (\rho_\phi + p_\phi)\,u_{\phi\,\mu}u_{\phi\,\nu} - p_\phi g_{\mu\nu}$ can be obtained, if $\nabla^\lambda\phi$ is timelike, by introducing the effective four-velocity of the scalar field fluid as [25]

$$u_{\phi\mu} = \frac{\nabla_\mu}{\sqrt{|\nabla^\lambda\phi\nabla_\lambda\phi|}}, \quad (100)$$

where $\nabla^\lambda\phi\nabla_\lambda\phi \neq 0$, and $u_{\phi\mu}u_\phi^\mu = \text{sign}\left(\nabla^\lambda\phi\nabla_\lambda\phi\right)$. The effective fluid pressure and energy density associated to the scalar field energy momentum tensor can be obtained as $\rho_\phi = T_{\mu\nu}^{(\phi)}u_\phi^\mu u_\phi^\nu$ and $p_\phi = T_{\mu\nu}^{(\phi)}h^{\mu\nu}/3$, where $h^{\mu\nu} = g^{\mu\nu} - u_\phi^\mu u_\phi^\nu$, and are given by [25]

$$\rho_\phi = \left[\nabla^\lambda\phi\nabla_\lambda\phi/2 - V(\phi)\right]\text{sign}\left(\nabla^\lambda\phi\nabla_\lambda\phi\right), \quad (101)$$

and

$$\begin{aligned} p_\phi &= \left[-1 + \text{sign}\left(\nabla^\lambda\phi\nabla_\lambda\phi\right)/2\right]\nabla^\lambda\phi\nabla_\lambda\phi \\ &\quad - \left[4 + \text{sign}\left(\nabla^\lambda\phi\nabla_\lambda\phi\right)\right]V(\phi), \end{aligned} \quad (102)$$

respectively. Therefore if the four-velocities of the dark matter fields, u_{DM}^μ and of the scalar field u_ϕ^μ satisfy the condition

$$u_{DM}^\mu \neq u_\phi^\mu, \quad (103)$$

the mixture of the non-comoving scalar field (dark energy) and dark matter can be described by a single

anisotropic fluid. Such a situation may have occurred during the inflationary, or post-inflationary (reheating) era of the evolution of the Universe.

The present model can be easily generalized for the case of a three component mixture, or for cases involving the presence of condensates and multi-component scalar fields. These possibilities, their impact on the cosmological evolution, as well as some observational implications, will be considered in future studies.

[1] P. A. R. Ade et al., Planck 2013 results. I, arXiv: 1303.5062 [astro-ph) (2013).

[2] P. A. R. Ade et al., Planck 2013 results. XVI, arXiv: 1303.5076 [astro-ph] (2013).

[3] P.A.R. Ade et al., Planck 2013 results. XXII, arXiv: 1303.5082 [astro-ph] (2013).

[4] P. A. R. Ade et al., Planck 2013 Results. XXIV, arXiv:1303.5084 [astro-ph] (2013).

[5] P. A. R. Ade et al., Planck 2013 Results. XXVI, arXiv:1303.5086 (astro-ph) (2013).

[6] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); T. Padmanabhan, Phys. Repts. **380**, 235 (2003).

[7] A. G. Riess et al., Astron. J. **116**, 1009 (1998); S. Perlmutter et al., Astrophys. J. **517**, 565 (1999); P. de Bernardis et al., Nature **404**, 955 (2000); S. Hanany et al., Astrophys. J. **545**, L5 (2000).

[8] C. Wetterich, Nuclear Physics **B 302**, 645 (1988); P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev **D 37**, 3406 (1988); R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); L. Amendola, Phys. Rev. **D 62**, 043511 (2000).

[9] A. Yu. Kamenshchik, U. Moschellai, V. Pasquier, Phys. Lett. B 511, 265 (2001); M. C. Bento, O. Bertolami, A. A. Sen, Phys. Rev. **D 66**, 043507 (2002).

[10] S. Capozziello, Int. J. Mod. Phys. **D 11**, 483 (2002); S. Nojiri and S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007); S. Capozziello and M. Francaviglia, Gen. Rel. Grav. **40**, 357 (2008); T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010); A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010); S. Nojiri, S. D. Odintsov, Phys. Rept. **505**, 59 (2011).

[11] R. Maartens and K. Koyama, Living Rev. Relativity **13**, 5 (2010).

[12] J. M. Overduin and P. S. Wesson, Phys. Repts. **402**, 267 (2004).

[13] S. Capozziello, V. F. Cardone and A. Troisi, JCAP **0608**, 001 (2006); S. Capozziello, V. F. Cardone and A. Troisi, Mon. Not. R. Astron. Soc. **375**, 1423 (2007); A. Borowiec, W. Godlowski and M. Szydlowski, Int. J. Geom. Meth. Mod. Phys. **4** (2007) 183; C. F. Martins and P. Salucci, Mon. Not. Roy. Astron. Soc. **381**, 1103 (2007); C. G. Boehmer, T. Harko and F. S. N. Lobo, Astropart. Phys. **29**, 386 (2008); C. G. Boehmer, T. Harko and F. S. N. Lobo, JCAP **0803**, 024 (2008); O. Bertolami, C. G. Boehmer, T. Harko and F. S. N. Lobo, Phys. Rev. **D75**, 104016 (2007); F. S. N. Lobo, arXiv:0807.1640 [gr-qc]; S. Capozziello, T. Harko, T. S. Koivisto, F. S. N. Lobo, and G. J. Olmo, arXiv:1212.5817 (2012).

[14] C. J. Copi, D. Huterer, D. J. Schwarz, and G. D. Starkman, Adv. Astron.**2010**, 847541 (2010).

[15] T. S. Koivisto, D. F. Mota, M. Quartin, and T. G. Zlosnik, Phys. Rev. **D 83**, 023509 (2011).

[16] M. Watanabe, S. Kanno, and J. Soda, MNRAS: Letters **412**, (2011), L83 (2011); J. Soda, Classical and Quantum Gravity **29**, 083001 (2012); P. Aluri, S. Panda, M. Sharma, and S. Thakur, arXiv:1210.3159 (2012); N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, arXiv:1210.3257 (2012); Z. Chang and S. Wang, arXiv:1303.6058 (2013).

[17] S. A. Appleby and E. V. Linder, Phys. Rev. **D 87**, 023532 (2013).

[18] P. S. Letelier, Phys. Rev. **D22**, 807 (1980); P. S. Letelier and P. S. C. Alencar, Phys. Rev. **D34**, 343 (1986).

[19] S. R. Oliveira, Phys. Rev. **D 40**, 3976 (1989).

[20] L. Herrera and N. O. Santos, Physics Reports **286**, 53 (1997).

[21] T. Harko and F. S. N. Lobo, Phys. Rev. **D 83**, 124051 (2011); T. Harko and F. S. N. Lobo, Astroparticle Physics **35**, 547 (2012).

[22] A. L. Maroto JCAP **0605** (2006) 015; A. L. Maroto, Int. J. Mod. Phys. **D 15**, 2165 (2006); J. B. Jimenez and A. L. Maroto, Phys. Rev. **D 76**, 023003 (2007); J. B. Jimenez and A. L. Maroto, JCAP **0903** (2009) 015.

[23] T. Koivisto and D. F. Mota, JCAP **0806** (2009) 018.

[24] C. Bonvin, R. Durrer, and M. A. Gasparini, Phys. Rev. **D73**, 023523 (2006); D. J. Schwarz and B. Weinhorst, Astron. Astrophys. **474**, 717 (2007).

[25] V. Faraoni, Phys. Rev. **D 85**, 024040 (2012).